Homework Set 3

(sect 1.5 – 1.8)

For the following systems, write the system first as a matrix equation, then write the augmented matrix that corresponds to the matrix equation. Solve this system and write the solution both as a vector and in parametric form.

1. $$\begin{cases} x_1 - 3x_2 + 7x_3 = 0 \\ -2x_1 + x_2 - 4x_3 = 0 \\ x_1 + 2x_2 + 9x_3 = 0 \end{cases}$$

2. $$\begin{cases} x_1 + 3x_2 - 5x_3 = 4 \\ x_1 + 4x_2 - 8x_3 = 7 \\ -3x_1 - 7x_2 + 9x_3 = -6 \end{cases}$$

3. $$\begin{cases} 4x_1 - 2x_2 + 7x_3 = -5 \\ 8x_1 - 3x_2 + 10x_3 = -3 \\ 3x_1 - 2x_2 - 2x_3 = 2 \end{cases}$$

4.
$$\begin{cases} 2x_1 + 5x_2 = 0 \\ 3x_1 + 6x_2 + 8x_3 = 0 \\ 4x_1 + 7x_2 + 9x_3 = 10 \end{cases}$$

5.
$$\begin{cases} 11x_1 - 3x_2 = 30 \\ -3x_1 + 6x_2 - x_3 = 5 \\ -x_2 + 3x_3 = -25 \end{cases}$$

6.
$$\begin{cases} -7x_1 - 48x_2 - 16x_3 + 3x_4 = 9 \\ x_1 + 14x_2 + 6x_3 - 2x_4 = -5 \\ -3x_1 - 45x_2 - 19x_3 + x_4 = 5 \\ x_1 - x_2 + x_3 = -1 \end{cases}$$

Determine whether the following vectors are linearly independent.

7. $\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$

8. $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$

9. $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$

10. $\begin{bmatrix} 1 \\ -3 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \\ 2 \end{bmatrix}$

11. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ -10 \\ -22 \\ -12 \end{bmatrix}$

Define the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Find $T(\boldsymbol{u})$ and $T(\boldsymbol{v})$ for given A, **u**, and **v**.

12. $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \end{bmatrix}, \ \boldsymbol{u} = \begin{bmatrix} -3 \\ 5 \\ -4 \end{bmatrix}, \ \boldsymbol{v} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

13. $A = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \ \boldsymbol{u} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}, \ \boldsymbol{v} = \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}$

Define the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Find a vector $\boldsymbol{x}$ such that $T(\boldsymbol{x}) = \boldsymbol{b}$. Is $\boldsymbol{x}$ unique for our choice of $\boldsymbol{b}$?

14. $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

15. $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

Determine whether the given transformation T is a linear transformation. If T is not a linear transformation, given an example that shows how it fails to be linear. Note: for these questions the vectors are row vectors instead of column vectors.

16. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the transformation defined by $T(x_1, x_2) = (x_2, 2x_1, -|x_1|)$.

17. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the transformation defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$.

18. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the transformation defined by $T(x_1, x_2, x_3) = (4x_1, x_1x_2)$.

19. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation defined by $T(x_1, x_2, x_3) = (x_1, x_2, 1)$.

20. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that reflects each vector $\boldsymbol{x} = (x_1, x_2, x_3)$ through the plane $x_3 = 0$ onto $T(\boldsymbol{x}) = (x_1, x_2, -x_3)$.

21. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that projects each vector $\boldsymbol{x} = (x_1, x_2, x_3)$ onto the plane $x_2 = 0$, so $T(\boldsymbol{x}) = (x_1, 0, x_3)$.

Use a rectangular coordinate system to plot $\boldsymbol{u} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, and their images under the given transformation T. (Make a separate and reasonably large sketch for each question.) Describe geometrically what T does to each vector $\boldsymbol{x}$ in $\mathbb{R}^2$.

22. $T(\boldsymbol{x}) = \begin{bmatrix} -.5 & 0 \\ 0 & -.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

23. $T(\boldsymbol{x}) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

24. $T(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

25. $T(\boldsymbol{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$